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# Topology optimization: a tool for the tailoring of structures and materials

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Is there a systematic way to minimize the weight of car and aeroplane parts? Is there a way to design materials that expand transversely when pulled? To design materials that shrink when heated? To design robots that are so small that they can be mounted on the head of a pin?

The answer to all four questions is affirmative; the method that solves the problems is called 'topology optimization'. This method is based on complex computer calculations. This paper describes the background of the method and shows a number of applications, ranging from the design of materials with 'exotic' properties over microscopic robots to the design of large-scale satellite structures.

**Keywords: optimization; numerical algorithms; finite-element analysis; material modelling; constitutive properties; mechanism synthesis**

## 1. Introduction

Imagine yourself driving down a highway in your new car. You enjoy the car's acceleration power, its spaciousness, its quietness, and, if you are ecologically oriented, its great fuel economy. You drive over the new highway bridge leading to the airport. You are on the way to pick up friends arriving on the new Airbus from New York.

You may not have thought about it before, but try to imagine how many hours engineers have spent on designing your new car, the highway bridge or the Airbus! It probably took several years of labour to design the bridge alone, not to mention the hundreds of human-years of labour required to design the car and the aeroplane.

Of course, engineers do not start from scratch when they begin to design a new car. They benefit from the experience that car engineers have gained throughout the century. They may even reuse the engine or the frame of an old car model. However, due to the ever-increasing desire for lower fuel consumption and increased driving comfort and safety, the engineers are faced with a dilemma. In order to decrease the fuel consumption they must decrease the weight of the car, but, on the other hand, increased driving comfort requires a bigger (and heavier) car and increased crash-worthiness may also require extra structural weight to build strength into the car frame. The same dilemma arises for the engineers of the aeroplane. The weight of the aeroplane should be minimized in order to save fuel and carry more passengers, but, at the same time, the aeroplane should be strong enough to withstand storms, turbulence and hard landings.





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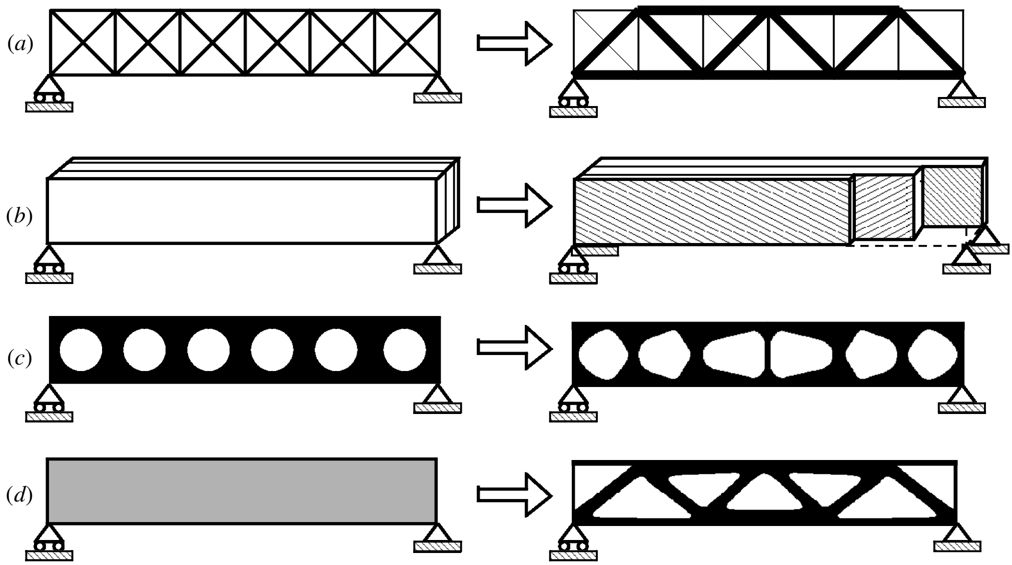


Figure 1. Four categories of structural optimization: (*a*) sizing optimization; (*b*) material optimization; (*c*) shape optimization; and (*d*) topology optimization. The initial problems are shown at the left-hand side and the optimal solutions are shown at the right.

Minimizing the weight of a structure while at the same time satisfying various requirements on structural response, cost, aesthetics and manufacturing is a complicated task. Experienced engineers may be able to come up with solutions that fulfil some of the requirements, but they will seldom be able to come up with the optimal structure. In order to both optimize the structure and meet the given requirements, the engineers must make use of computer programs.

A computer program for the optimization of mechanical structures will typically consist of an *analysis module* and an *optimization module*. The analysis module is used to calculate the structural response. It can, for example, be used to calculate the maximum deflection or the resonance frequency of the structure. The analysis module is also used to perform a *sensitivity analysis*. This corresponds to calculating the change in the structural response for a small design change. Based on the sensitivity analysis, the optimization module calculates a change in the structural design that improves the response. Typically, the optimal design is not achieved after only one optimization step. Instead, the procedure, consisting of the analysis, the sensitivity analysis and the optimization step, is repeated several times. After a number of iterations, the design cannot be further improved and an optimal structure has been reached. The development of efficient computer programs for the optimization of structures is a very active area of research. The research area is called *structural optimization*.

Structural-optimization methods can be divided into four main categories. As an example, consider the beam structure sketched in figure 1. The goal is to design the beam such that the stiffness is maximized for a given weight. The differences between the four structural-optimization categories mainly consist of the definition of the *design variables*. The design variables are the parameters that can be changed during the optimization process.

### (a) Sizing optimization

A simple sizing-optimization problem is shown in figure 1*a*. In the sizing-optimization problem, the layout of the structure is prescribed; in this case, it is a truss structure consisting of 31 truss elements. The cross-sectional area of each element is a design variable. The truss structure is optimized by finding the areas of the individual truss elements that maximize the stiffness of the truss structure for a given total weight. Sizing optimization is the simplest way of doing structural optimization.

### (b) Material optimization

Instead of building the beam as a truss structure, it can be built as a layered fibre-composite. The goal here is to find the material composition that optimizes the stiffness of the beam. In the beam design case, the design variables are the orientations and thicknesses of the individual layers of the composite as sketched in figure 1*b*.

### (c) Shape optimization

An intuitive way to save weight is to drill circular holes in the structural component. However, circular holes are not structurally efficient. Stress concentrations may be high along the edges of the holes and may cause the structure to break when loaded. The structure may be improved using *shape optimization*. In this case, the design variables are parameters that change the shape of the holes. The procedure is illustrated in figure 1*c*.

### (d) Topology optimization

The sizing-optimization, material-optimization and shape-optimization methods all consider the optimization of structures with fixed *topologies*. The word *topology* originates from the Greek word ‘topos’, which means landscape or place. In other words, a structure with a fixed topology can be said to have a fixed ‘landscape’. In the case of the sizing-optimization problem, the number and connectivity of the bar elements were fixed. In the material-optimization problem, the structure was fixed to be a simple beam. In the shape-optimization case, the number of holes in the structure was fixed at six. It is quite obvious that the number of truss elements in the first structure and the number of holes in the last structure will influence the response and the weight of the structure significantly. This means that the topology should be a variable in order to optimize its behaviour.

The above categorization of structural-optimization methods is rather idealized. Often, structural-optimization methods consist of mixtures of the categories. In fact, the topology-optimization method combines all four methods. The method not only finds the optimal number of holes in a structure, but also finds the optimal shape of the holes and the optimal areas of the bars making up the structure. In special cases, topology-optimization methods may also be used to find the orientations and thicknesses of fibre layers in the optimal structure.

When the computer-based topology-optimization method was first introduced by Bendsøe & Kikuchi (1988), it was intended for the minimum-weight design of structural components. Since then, the topology-optimization method has gained




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widespread popularity in academia and industry, and it is now being used to reduce weight and optimize performance of automobiles, aircraft, space vehicles and many other structures. Recently, the method has also been applied to a number of other design problems. Examples are the design of tailored 'exotic' materials with counter-intuitive properties, such as negative Poisson's ratios (materials that expand transversely when pulled) and negative thermal-expansion coefficients (materials that shrink when heated). Other applications include the design of transducers for underwater sound detection, car parts for crash-worthiness, medical implants and Micro-ElectroMechanical Systems (MEMSs) for use in hearing aids, air-bag sensors and micro-robots.

A short introduction to the topology-optimization method and its mode of operation will be demonstrated with examples in § 2. Applications of the topology-optimization method to the design of a satellite structure, materials with negative Poisson's ratio and negative thermal-expansion coefficients, and to the design of a microscopic robot will be discussed in §§ 3–5.

## 2. The topology-optimization method

The development of the theory behind the topology-optimization method dates back to the work of Michell (1904), who set up the conditions for optimality of load-carrying structures. Since Michell's pioneering work, engineers and mathematicians have worked on refining the theories. Based on the theoretical work, Bendsøe and Kikuchi founded the computer-based topology-optimization algorithm.

The topology-optimization method solves the most general structural-optimization problem of distributing a given amount of material freely in the design space such that performance is optimized. The design of an aeroplane floor support beam is used to illustrate the procedure (figure 2).

The first step in the topology-optimization algorithm is definition of the *design domain*. The design domain or the space that the structure is allowed to occupy can be restricted for various reasons. For example, the height of the design domain for the aeroplane beam from figure 2*a* is restricted by the passenger cabin from above and by the baggage space from below.

The next step in the topology-optimization algorithm is to define the *load and support conditions* that will influence the design of the structure. In the case of the aeroplane beam, the main load comes from the weight of seats and passengers. If the beam is situated between the wings, it will also be subject to a load from the weight of the wings and the aerodynamic forces acting on them. The support conditions are defined by the points where the beam is attached to the aeroplane fuselage. Simplified load and support conditions for the aeroplane beam are shown in figure 2*b*.

When the design domain has been specified and the load and support conditions have been defined, the response of the structure must be analysed. The analysis of the structural response is carried out by dividing the structure into numerous small elements called *finite elements*, as seen in figure 2*c*. While the response of a geometrically complex structure can be difficult to calculate, very simple equations can be set up for a small and geometrically simple finite element. By collecting the simple equations for each element into one big system of equations, the response of the whole structure can be calculated. The method of dividing the structure into small simple elements and finding the global response by combining the simple equations

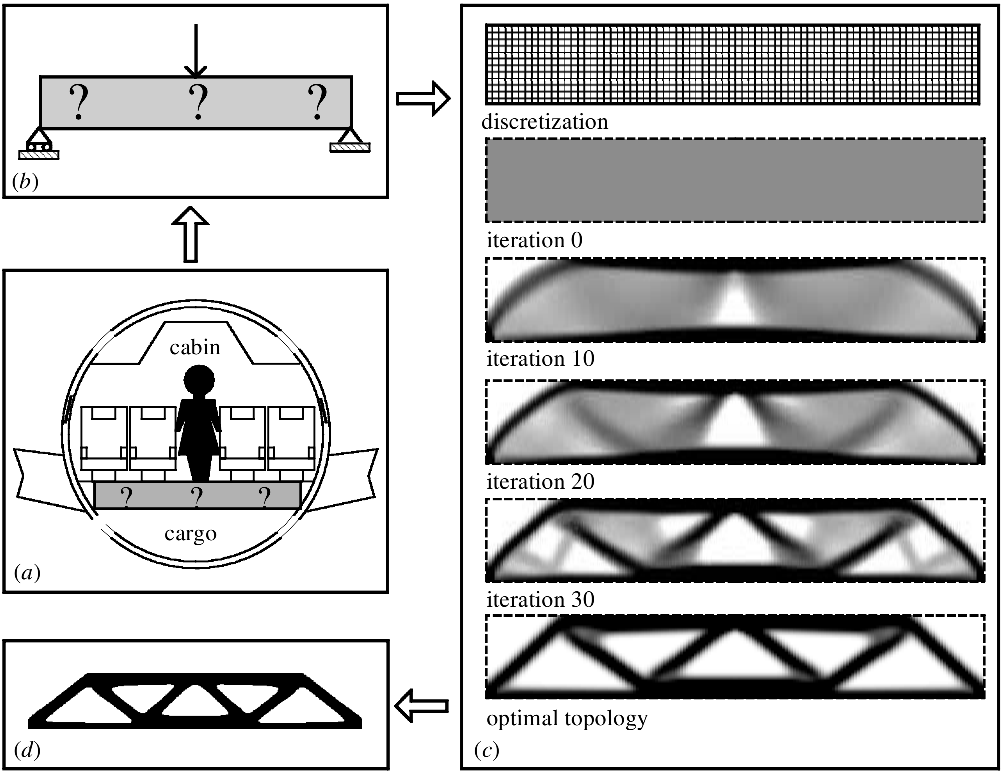



Figure 2. Design of an aeroplane floor support beam using the topology-optimization method. (*a*) Definition of the design domain (grey area); (*b*) definition of load and support conditions; (*c*) discretization and iteration history; and (*d*) final post-processed design.

is called *finite-element analysis*. Using finite-element analysis, one can determine structural responses such as stiffness, resonance frequency, maximum stress level, impact and thermal responses, and many other structural characteristics.

The finite-element analysis is the basis for improving the structural response. One result of finite-element analysis is the stress distribution in the structure. Some parts of the structure may be highly stressed and some parts may only experience low stresses. An intuitive way of optimizing the structure is to add material to areas with high stresses and to remove material from areas with low stresses. In fact, many optimization algorithms are based on this intuitive idea. However, this approach will fail for more complex load conditions. Instead, a sensitivity analysis must be carried out. The sensitivity analysis determines the changes in structural response for a small change in each design variable. Based on the sensitivity analysis, the optimization module determines the change in design variables that improves the response as much as possible.

As discussed in § 1, structural-optimization methods differ from each other in the way that the design variables are defined. In the case of the topology-optimization method, *the design variables describe the density of material in each finite element.* In other words, one may consider the design domain as a black and white television screen divided into a lot of small pixels (or finite elements). By turning material on and off in each pixel, one can produce a picture of the optimal structure. In practice, grey pixels corresponding to elements with porous material are allowed during the design process, but only black and white pixels corresponding to material or no material elements are left in the final design.

The iterative design procedure is illustrated in figure 2. Before the first iteration, the beam is discretized by 5400 finite elements. The load is assumed to be a single load at the centre of the beam. The beam is supported in the lower left- and right-hand corners and the available amount of material is distributed evenly in the design domain. Then the iterative procedure is started. It is seen that after 10 iterations some material has been moved to the upper and lower parts of the beam where the stresses are larger. After 30 iterations the optimal structure is roughly outlined, and after 50 iterations the optimal structure has been found.

As described above, the implementation of the topology-optimization method sounds fairly simple. However, different numerical and theoretical challenges arise during the process. These challenges shall not be described in detail here, but one of them should be briefly mentioned, namely the problem of *mesh dependency*. The mesh-dependency problem can be described as follows: the more elements that are used to discretize the structure, the more details will appear in the optimal structure. In fact, the very best structure will be a structure consisting of an infinitely fine grid of closely spaced beams. A structure with a very complex grid of beams is in most cases impractical for economical and geometrical reasons. The structure will be expensive to manufacture and, at least for the aeroplane beam example, it will be impossible to run control cables and electric cables through the holes. To prevent the appearance of very small structural details, one can use filtering or perimeter-control techniques.

### (*a*) *Michell's optimal structures*

As mentioned in § 1, Michell was the pioneer of structural optimization. In his paper from 1904, he set up conditions for optimality of simply loaded structures. One of the conditions is that: 'A more general class of (optimal) frames... consist of those whose bars,... form curves of orthogonal systems'. Using this simple rule, Michell was able to construct several optimal structures, two of which are shown in figure 3. In the first example, a single vertical load is to be suspended between two supports. Using his basic conditions and geometrical intuition, Michell came up with the optimal design shown in figure 3*a*. Another of Michell's examples is shown in figure 3*b*. Here, a single load has to be transferred to a circular support. Common to both examples is that all bars making up the optimal structures meet each other at right angles.

A way of testing a computer-based topology-optimization program is to solve Michell's basic examples and compare the resulting topologies with his theoretically developed designs. The topology-optimized solutions to Michell's two examples are shown in parts (*c*) and (*d*) of figure 3, respectively. It is seen that the numerically obtained topologies are very similar to Michell's predictions.

Now one may ask: why spend a lot of work on the development of a computer program if one can read a century-old paper and find the solution to the design problem? The answer is that Michell's method only works for very simple load conditions. The

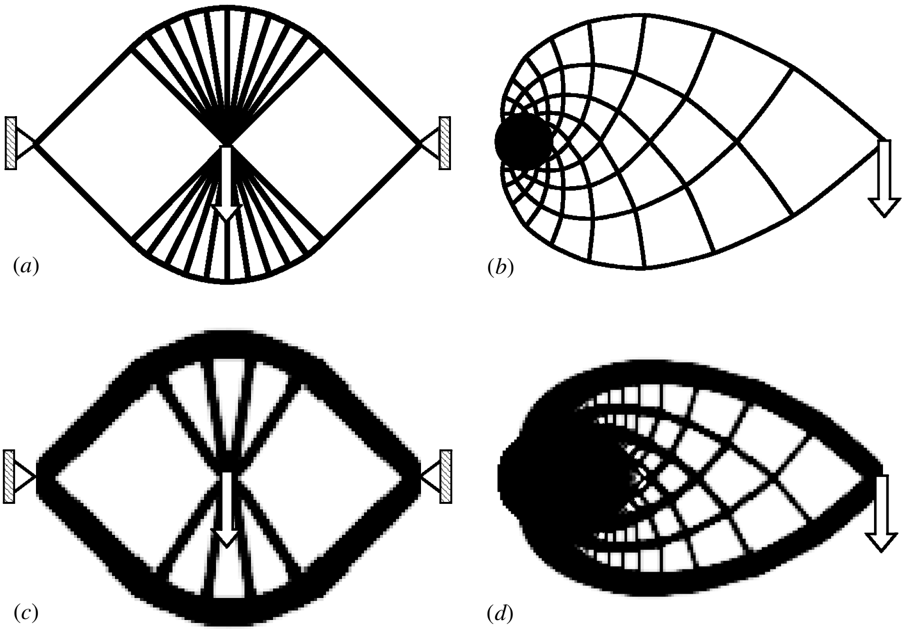



Figure 3. Michell's optimal layouts: (*a*) Michell's optimal design for a single load with two supports; (*b*) Michell's optimal design for a single load with a circular support; (*c*) and (*d*) topology-optimized solutions.

analytical method fails for multiple load conditions, for dynamically loaded structures, and for problems involving modelling in multiple physical domains. Examples of structures that cannot be designed using analytical methods will be given in the following sections.

## 3. Design of a satellite

Skilled engineers may be able to come up with efficient designs for geometrically simple structures. An example of a geometrical complexity that goes beyond the abilities of engineers is the design of a small satellite structure.

The Danish government is sponsoring a small satellite programme with the aim of launching a satellite with scientific mission goals every fifth year. One of the proposals for the next launch is a small satellite that can investigate the physics behind gamma-ray bursts appearing in distant galaxies. Physicists and astronomers are disagreeing on the source of the gamma-ray bursts, but there is agreement that the gamma-ray bursts release energies that are bigger than any previously known energy releases. An average of one burst can be detected per day with the strongest signal appearing in the first few minutes after the burst. The release of gamma rays decays to a non-detectable signal in *ca.* 24 h. The satellite should, therefore, be able to detect the burst and turn its telescope towards the source as fast as possible.

To solve its mission, the satellite will be equipped with four wide-angle cameras that can search the whole space for gamma-ray bursts. Once one of the cameras

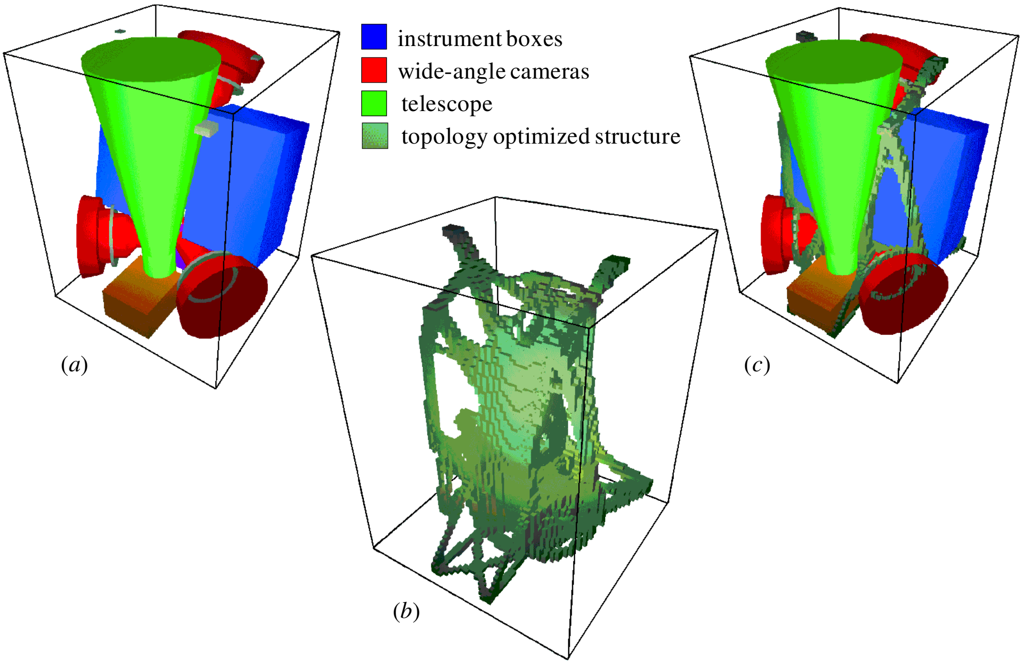



Figure 4. Design of a small satellite. (*a*) Design domain and instrumentation; (*b*) topology-optimized support structure; and (*c*) support structure with instrumentation.

detects a burst, the satellite will orient its telescope towards the source and record the signal. In addition to the four cameras and the telescope, the satellite will be equipped with electronic instruments for controls and communication, batteries and solar panels. The size of the satellite is limited to $60 \times 60 \times 80\ \mathrm{cm}^3$ and the weight is limited to 80 kg. The small size of the satellite makes it possible to launch it as a 'secondary' payload, which is economically favourable.

A way to mount the cameras, telescope and electronic boxes in the satellite is shown in figure 4*a*. The problem is how to design a support structure that weighs less than 12 kg, yet is strong enough to carry the instruments during launch. Furthermore, it should be possible to attach two hooks to the top of the structure for ground handling.

The design problem is very well suited for topology optimization. The design domain is a box-like structure in which material can be distributed everywhere, except for the space taken up by the instruments. The structure is supported by a circular ring attached to the launch rocket, and the main load case comes from the $15g$ acceleration force experienced during launch. Two other load cases simulate the sideways vibrations and a fourth load case simulates the ground handling. Finally, the resonance frequency of the whole satellite should be higher than 35 Hz.

The design domain is discretized using 288 000 cubic finite elements. The optimal design is shown in figure 4*b*. The computation took two days on a powerful workstation. It is evident that the structure is truss-like and supports all the instruments. Another view of the satellite structure with instruments is shown in figure 4*c*. The structure requires some post-processing.

## 4. Design of materials

> Any material is a structure if you look at it through a microscope with sufficient magnification.

This statement describes the philosophy behind the application of the topology-optimization method to the design of extremal materials.

Any material, be it foam, wood, bone or metal, has a microstructure if looked at through a microscope. The topology of the microstructure and the material composition determines the properties of the material, so why not use a method that was originally intended for the design of large-scale structures for the design of material microstructures to obtain materials with optimal properties? Through topology optimization of material microstructures, one can tailor new materials with improved properties, and, as will be shown in this section, materials with extreme or counter-intuitive properties.

Many materials have a periodic microstructure. An example is honeycomb material that consists of a repeated pattern of hexagonal cells. The mechanical behaviour of a honeycomb material can be analysed by studying just one of its cells, the *base cell*. The base cell is the smallest repetitive unit of the material. If the topology of the base cell is changed, the properties of the whole material will change. In the same way, material with optimal properties can be obtained by optimizing the topology of the base cell.

As for the topology optimization of large-scale structures, the topology of material microstructures is initiated by discretizing the base cell by finite elements and analysing the properties by finite-element analysis. In the material-design case, the base cell is tensionally loaded in different directions to find the stiffness and other properties of the material. Again, based on the sensitivity analysis, the optimization module determines the redistribution of material that will optimize the objective function.

To demonstrate the effect of material design, the aeroplane floor support beam from figure 2 is revisited. Instead of having the density of material in each element as a design variable, the material microstructure in each element is now a design variable. This means that the microstructure should be optimized for each point in the structure. This will result in a very stiff beam but also in a beam that will be very costly to manufacture.

The result of the optimization process is shown in figure 5*a*. The optimal beam now consists of regions with solid material (black) and regions with porous (grey), topology-optimized microstructures. The optimal beam is 10% lighter than the beam from figure 2*d*, but it is more difficult and more expensive to manufacture and control cables for the aeroplane cannot be led through it.

Nature also uses the principle of optimizing the point-wise material properties of structures. If a human bone is cut in two halves, it can be observed that the outer parts of the bone consist of almost solid bone, whereas the inner parts of the bone consist of porous microstructures (see figure 5*b*).

The study of bone structures and bone adaption is part of the research area called biomechanics. Many methods used in the study of bone evolution can be directly applied to structural optimization and vice versa. However, the exact connection between bone evolution and applied loads has not yet been discovered; apparently,

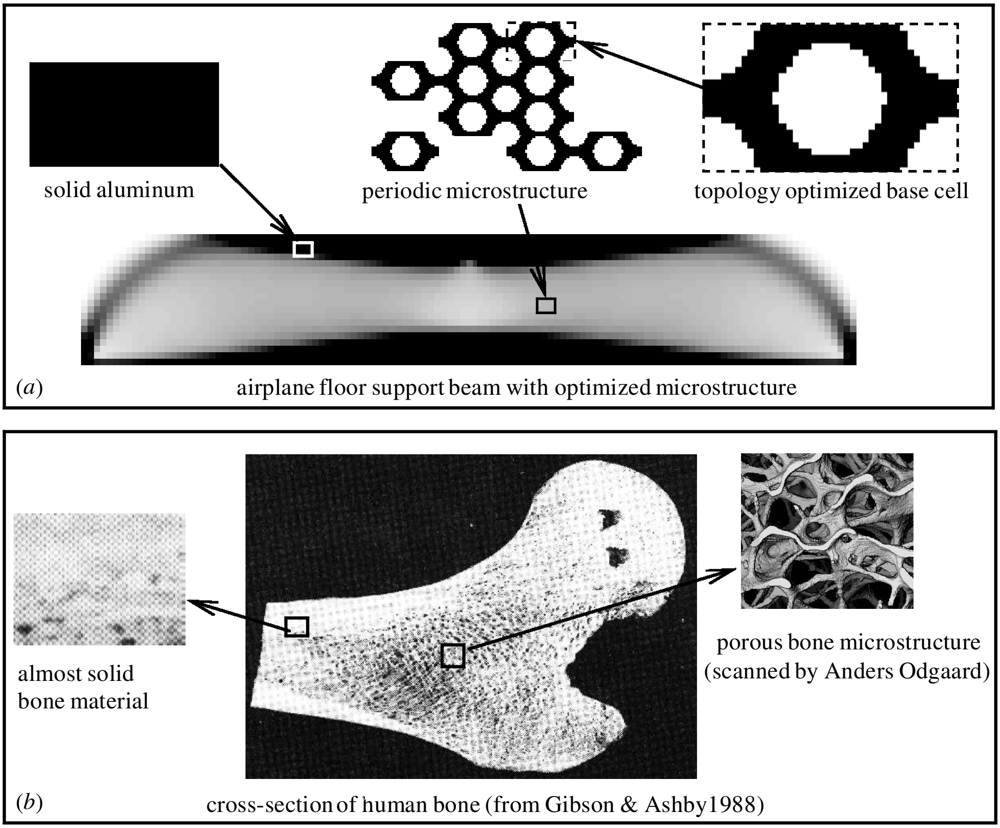



Figure 5. Design of the optimal beam. (*a*) The optimal beam consists of regions with solid material (black regions) and regions with topology-optimized porous microstructures (grey regions). (*b*) A cross-section of a human bone shows that the bone structure has solid and porous regions as well.

the physics of the human body is (still) more complex than an aeroplane floor support beam.

### (*a*) *Design of a negative Poisson's ratio material*

All naturally occurring materials shrink transversally when pulled. Imagine a rubber band: the more you pull it, the thinner it gets. The ratio between the transversal shrinking and the longitudinal elongation is called *Poisson's ratio*. The Poisson's ratios of all naturally occurring materials are positive, which means that they shrink when pulled.

Is it possible to build a material with negative Poisson's ratio, which means that it gets fatter when pulled? Intuition probably tells us no, but in fact topology optimization has the answer and it is affirmative!

Using the material topology-optimization algorithm to minimize the Poisson's ratio of the material results in a microstructure as seen in figure 6*a*. The material has negative Poisson's ratio. The mechanism behind this counter-intuitive behaviour is that it 'unfolds' when pulled, as illustrated in figure 6*b*.

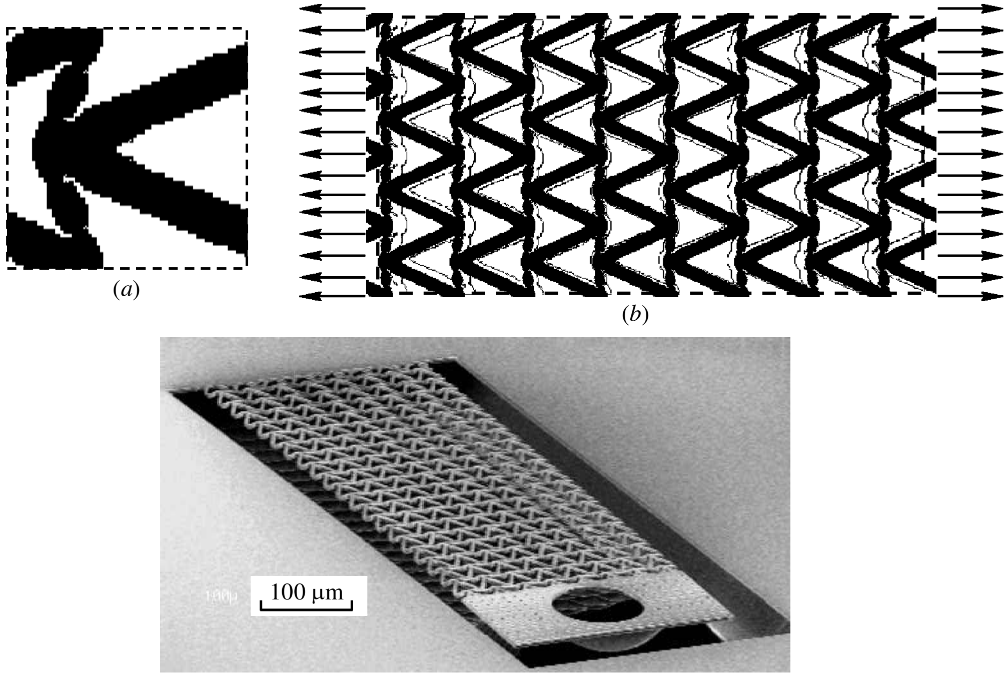


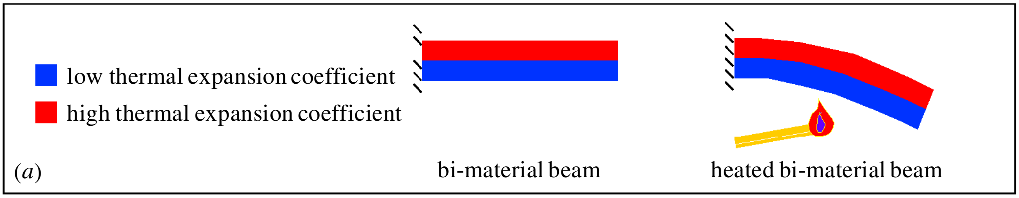

Figure 6. Design of a material with negative Poisson's ratio. (*a*) Topology-optimized base cell with negative Poisson's ratio. (*b*) Elongation of negative Poisson's ratio test beam; note that it becomes fatter when pulled (the dashed background lines show the undeformed structure). (*c*) Micromachined testbeam fabricated at the Microelectronics Centre (MIC), Technical University of Denmark (length 1 mm).

An obvious question is: what is a negative Poisson's ratio material good for? There are many answers to this question but one of the best applications is for fasteners. It is easy to push a fastener made of a negative Poisson's ratio material into the wall since it gets slimmer when pushed in. On the other hand, once it is in the wall it is difficult to pull out since it expands when pulled.

A test beam made from the negative Poisson's ratio material was built using laser-micromachining techniques at the Microelectronics Centre (MIC) at the Technical University of Denmark (DTU). A scanning electron micrograph of the beam is shown in figure 6*c*. The beam is 1 mm long and each cell is 50 μm square. It is impossible to see the microstructure with the naked eye, so the beam was tested using a microprobe and the response was measured with a microscope. The Poisson's ratio was measured to be $-0.9$, close to the theoretically predicted value.

### (*b*) *Design of a material with negative thermal-expansion coefficient*

Another example of a material with 'exotic' properties is a material with a negative thermal-expansion coefficient. Most naturally occurring materials have positive thermal-expansion coefficients, which means that they expand when heated. Imagine for example a railroad track. In extremely warm weather the railroad track may expand so much that it bends into an S-shape and the train derails. The



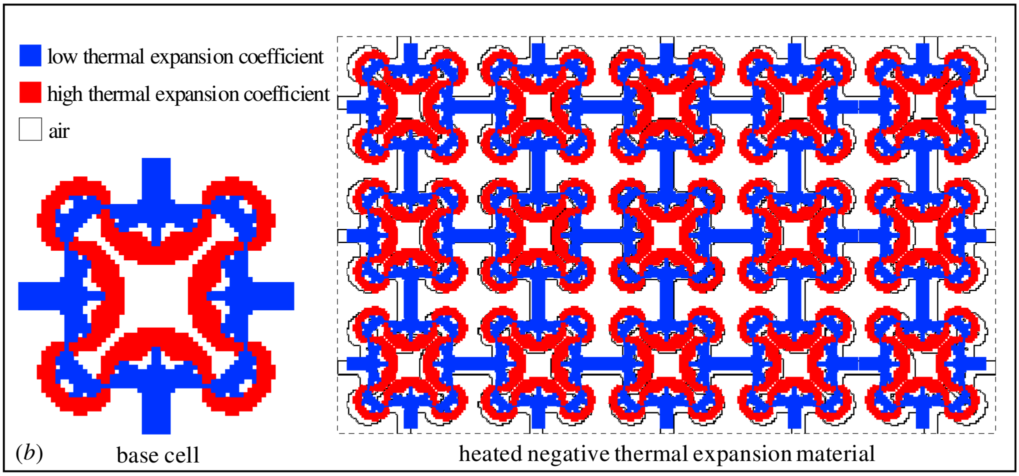




Figure 7. Design of a material with negative thermal-expansion coefficient. (*a*) The bi-material principle; and (*b*) topology-optimized two-material microstructure that contracts when heated (the dashed lines denote the undeformed structure).

phenomenon is also found in bridges, where the length can vary several metres between summer and winter and special length equalization segments have to be constructed to prevent gaps from appearing in the bridge. Thermal expansion is also a big problem in space. The temperature difference between the sunny and the shady side of a space structure may be several hundred degrees, and this may cause large space antennas to distort and thereby lose their ability to send and receive signals.

Because of all these problems with thermal expansion due to temperature changes it would be nice to have access to materials with either zero thermal-expansion coefficients or materials with negative thermal-expansion coefficients that could neutralize the positive expansion of normal materials.

In order to design a material with negative thermal-expansion coefficient one can make use of the *bi-material effect*. If one makes a sandwich beam of two materials with different thermal-expansion coefficients, the beam will bend towards the side with the lower thermal-expansion coefficient when heated. The principle is illustrated in figure 7*a*.

The material topology-optimization algorithm is now modified to include the distribution of two different material phases in the base cell, as shown in figure 7*b*. The red phase has a high thermal-expansion coefficient and the blue phase has a low (but still positive) thermal-expansion coefficient. Minimizing the effective thermal-expansion coefficient of the microstructure, one obtains the topology shown in figure 7*b*. The resulting microstructure has a negative thermal-expansion coefficient.

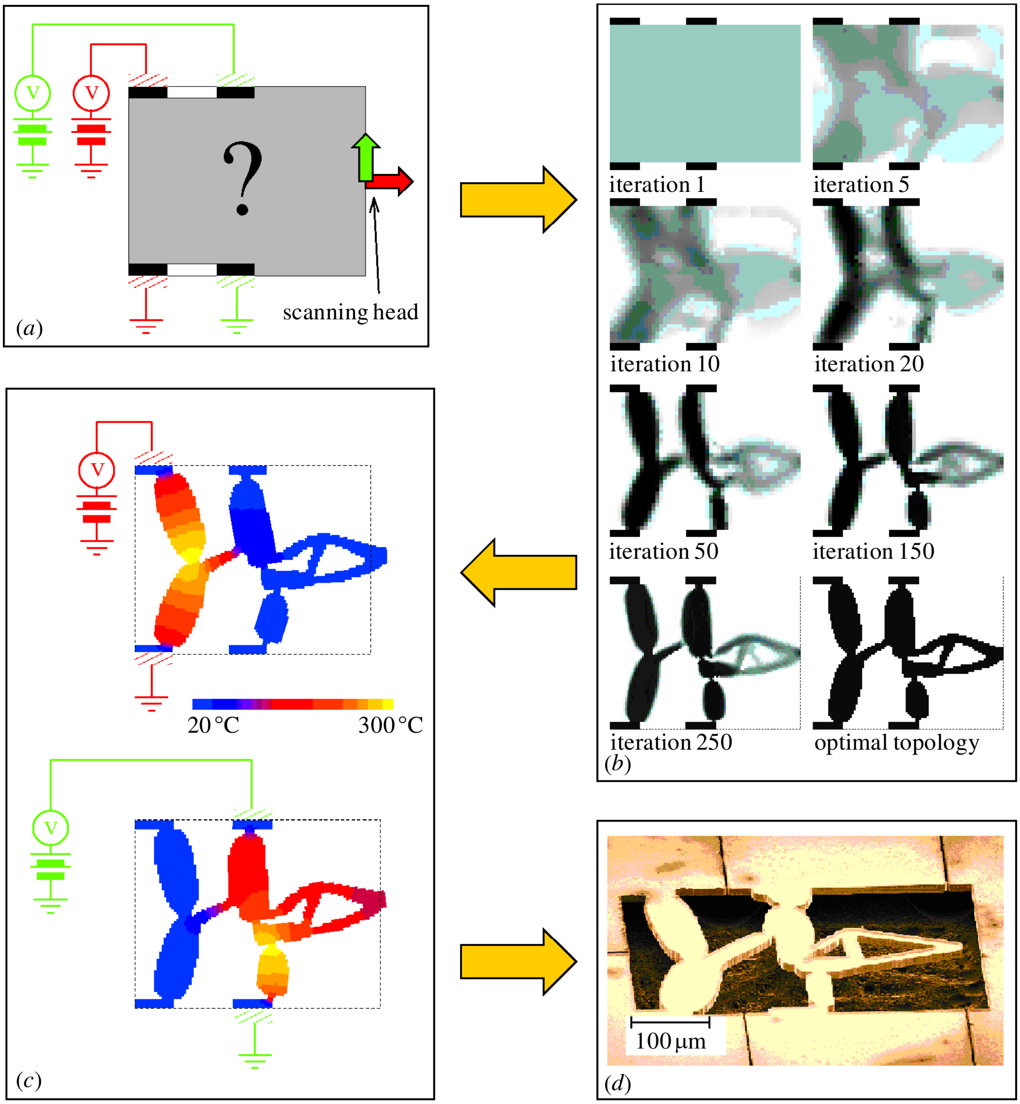



Figure 8. Design of a microscopic two-degrees-of-freedom actuator. (*a*) Definition of the design domain and electric load conditions; (*b*) iteration history; (*c*) simulation of the heat distribution and displacement of the optimized actuator; and (*d*) micromachined actuator fabricated at MIC, DTU.

Studying the optimal topology, one notices that the microstructure consists of several small bi-material beams that, in an intricate way, make the periodic structure contract when heated, even though the materials it is built from expand when heated (see figure 7).

## 5. Design of micro-robots

A new application of the topology-optimization method is in the design of MicroElectroMechanical systems (MEMSs). These are microscopic mechanical devices coupled with electronic circuits. The small size of MEMSs give them several advantages over conventional-size mechatronic systems. For example, MEMSs can be used as flow sensors, which, due to their small size, do not interrupt the flow. They can also be used to remove blood clots inside the human body, and as hearing aids so small that they can be implanted in the ear.

MEMSs are manufactured using etching and deposition processes known from the semiconductor industry. The base material is a silicon chip. On top of the silicon chip, different materials can be deposited. An outline of the mechanical device is etched in the top layer. Using the top layer as a mask, other etching agents are used to etch through lower-lying layers and finally another etching agent releases the device from the silicon chip. By varying the processes, quite complex mechanical devices can be built and integrated with electronic circuits on the same chip.

The small size and the manufacturing techniques for MEMSs do not allow for assembly processes, hinges and bearings, known from conventional-size mechanical systems. *MEMSs must be etched out of one piece of material.* The mobility of the mechanical systems must therefore come from bending in parts of the structures. Mechanisms that gain their mobility from bending are called *compliant mechanisms*. Design of compliant mechanisms is a complicated task, but it can be efficiently solved using the topology-optimization method.

In the following, the design of a microscopic two-degrees-of-freedom actuator will be used to demonstrate the application of the topology-optimization method to MEMS design.

As a future application of MEMSs, researchers are developing techniques to store information by moving atoms from one position to another on a microchip. In this way, it will be possible to store information that currently requires a large hard disk on a few square millimetres. To write and read from this small area, a microscopic pickup must be moved over the surface by a miniature robot arm. Two different electrical inputs to the robot should move it in two directions over the surface. The design problem thus consists of converting two independent electrical signals into two independent mechanical outputs.

Different actuation principles can be thought of but one of the simplest, and the one that will be used here, is the actuation by *Joule heating* of the robot. The principle of Joule heating is the following: if an electric field is applied over a piece of metal, it will heat up due to the electrical resistance. When the metal is heated, it will expand due to the positive thermal-expansion coefficient. The expansion gives the desired actuation. Unfortunately, the expansion due to Joule heating is quite small. However, the topology-optimization method can be used to amplify the expansion, and, at the same time, obtain the desired two-degrees-of-freedom output.

The design domain for the actuator is shown in figure 8*a*. The red electrical input must result in a horizontal movement of the output point, and the green electrical input must result in a vertical movement of the output point. The finite-element analysis of the actuator response now involves the simulation of the electric, thermal and elastic behaviour of the structure. However, the optimization process is the same as before. The evolution of the robot is illustrated in figure 8*b*. The output from the electrothermomechanical finite-element simulation of the robot is shown in figure 8*c*. It is seen that the red electrical input heats up the left part of the robot and makes it move horizontally. Equivalently, the green electric input heats up the centre part of the robot, and, due to an offset, the thermal expansion makes the output point move in the vertical direction.

The micro-robot has been built and tested at MIC, DTU. An electron micrograph of the tested device is shown in figure 8*d*. Note the size—the dimensions are $500 \times 400\ \mu m^2$ and the thickness is $20\ \mu m$—the robot is small enough to be mounted on the head of a pin!

## 6. Perspectives

This paper has described a relatively new computational method called topology optimization. Examples of its application to structural design, material design and microscopic-robot design have been given. The examples, however, only cover a few of the exciting applications of the method, some of which remain to be explored. It is easy to imagine that in the future, the method will be applied to every design problem where the use of material is limited and the response should be optimized. Using the topology-optimization method as a tool in the design process, engineers can also save considerable time and reduce costs in the prototyping process, which are both important factors in a competitive industry.

The reception of topology-optimized structures has not always been favourable. When it was first introduced more than a decade ago, the technique met with scepticism: the optimal topologies were said to be too costly to manufacture, or, worse still, not possible to manufacture at all. Meanwhile, manufacturing methods have caught up with the theory and made it possible to manufacture even very complex geometries using computer-controlled milling machines, stereolithography methods and laser micro-machining processes.

Taking the latest advances in topology optimization and manufacturing methods into account, the day is not far off when the engineer can specify the loading and working conditions of a structure and have a working prototype finished in a matter of hours, or when the engineer prints a block of custom-designed material with tailored thermoelastic properties on his desktop solid free-form processor.

## Further reading

In the last decade, several hundred papers have appeared on the topic of topology optimization. The reader is referred to a book by Bendsøe (1995) and the references therein. Selected papers and books (see, for example, Gibson & Ashby 1988; Sigmund 1997; Sigmund & Torquato 1996) with more details on the topics covered in this article are listed in the References.

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